GAME THEORY AND CUMULATIVE VOTING IN ILLINOIS

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Since the theory of games was first made widely available (von Neumann and Morgenstern, 1944), with application to economic behavior, its use has been suggested in many other areas, from the global (Kaplan, 1957) to the individual (Simon, 1956).

Its contribution to substantive knowledge in the empirical sciences, however, has been modest, and Luce and Raiffa (1957) judge that its use has been greater in applied mathematics. The area of political behavior—despite the apparent applicability of the notion of conflict of interest—is similarly lacking in studies, although notable exceptions exist in Shapley and Shubik (1954) and Luce and Rogow (1956).

Many of the previous studies have taken the form of defining a situation in terms of game theory and prescribing the proper behavior for a given set of conditions. In the present study, however, it was possible not only to specify a simple game theory model, but to evaluate it in a large number of actual cases. The following sections describe a voting behavior being modeled, the game theory model employed, an empirical test of the model, and implications of the results.

THE VOTING SITUATION

Voting for representatives for the Illinois General Assembly proceeds in a manner which is politically unique (although common in voting for corporate boards of directors). (See Glaser (1959) for an application of game theory to cumulative voting in this latter context.) The system, "cumulative voting," is intended to secure minority representation and does so by providing multiple member constituencies and allowing individuals to "cumulate" their votes on fewer than the total number of candidates to be elected.

In the case of cumulative voting in Illinois, three representatives are elected from each district, and each voter has three votes, which he may distribute 3-0, 2-1, $1\frac{1}{2}$-$1\frac{1}{2}$, or 1-1-1, among the candidates. Each party may nominate for the general election one, two, or three candidates, and the <u>number</u> to be nominated is decided upon and announced prior to the primary. This decision is made more or less independently by separate three-man committees elected by members of each party, with the voters in the primary election then determining <u>who</u> the candidates shall be.

The committee's decision is made under uncertainty as to the percentage of the vote which the party will receive, and often, though not always, the number of candidates which the other party will nominate. It is the behavior of this committee in arriving at a decision in the face of uncertainty which is being examined. In particular, the "rationality" of this decision in terms of its maximizing the number of the party elected is investigated by reference to the theory of games.

Previous studies of cumulative voting (Moore, 1919; Hyneman and Morgan, 1937; and Blair, 1960), while giving major attention to other aspects, have noted that occasionally the majority party fails to nominate two candidates and thereby loses the second seat it could otherwise fill. (As it turn out, however, by far the most frequent source of loss is when a party with a 75% majority nominates only two rather than three candidates.) Hence, in order to systematically examine the behavior of the nominating committee, the present study postulates, for the process of deciding upon the number of candidates, a model, based upon the theory of games, and described in the next section.

THE GAME THEORY MODEL

The behavior of the two nominating committees —one for each party—may be viewed as a two-person game in which the payoff is the number of candidates elected. The game is essentially zero-sum, in that positions not filled by one party are filled by the other. Each party has three strategies, namely to nominate one, two, or three candidates (the theoretical alternative of nominating none may be eliminated from consideration since it is never optimal and never employed).

Applicability of Model

The theory of games assumes that each player (1) knows all the rules of the game, i.e., the payoff matrix, (2) has a preference ordering of the payoffs, and knows that of his opponents, and (3) expresses his preference ordering in selecting strategies, i.e., he acts to maximize expected utility. As Luce and Raiffa (1957) point out, the third condition may perhaps best be taken as tautological, being simply a description of "preference ordering." The alternative is to determine preference orderings independently and use these to test the postulate.

In this application—unlike many experimental trials of the theory of games—the stakes are so considerable that there is good reason to assume a preference ordering. At the least, electing one candidate is not preferred to electing two candidates, and neither is preferred to electing three candidates. The assumption of a known payoff matrix is also met unusually well in this case, for both the available strategies and the outcomes for different combinations of strategies are completely prescribed and known to both parties. Hence, among situations involving actual political behavior, the case of cumulative voting appears particularly appropriate and specifically overcomes many of the problems raised by Deutsch (1954) in the application of game theory to politics.

Factors in the Committee's Decision

In determining the number of candidates to nominate, the committee acts under uncertainty as to (1) the percentage of the vote their party will receive, (2) how it will be divided among their candidates, and (3) how many candidates the other party will nominate. The division of the vote among the various candidates of the same party,

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however, in practice is usually very even, reflecting perhaps a combination of voter indifference, party discipline, and the effect (in machine voting) of the party lever, which results in an equal division, whereas to split one's vote unevenly requires further manipulation. In any event, given a rational opponent, an equal division of the votes among the candidates is always best. Otherwise, as has happened in a small percentage of elections, a party barely in the majority may find its less preferred candidate running behind both the equally preferred minority party candidates. Consequently, the formulations which follow assume that each party's vote is divided equally among all candidates of that party. (Actually, as will be made apparent later, the most desirable situation—although very difficult to attain—would be the ability to maintain a very small, but highly reliable, difference among the candidates.

The Payoff Matrices

The two remaining uncertainties may be dealt with by considering, for any given distribution of the vote, what the payoffs are in terms of number of candidates elected for any particular combination of strategies. It turns out, in fact, that there are only six different payoff matrices, one for each of the following ranges of the vote for the first party: 0-25%, 25-40%, 40-50%, 50-60%, 60-75%, 75-100%. These six matrices are given in Table 1, with the payoff the number of candidates elected by party A.

Table 1

Number Elected by Party A, by Percentage of Vote

<u>0-25% A</u>

A Nominates	B Nominates 1	2	3
1	1	1	0
2	2	1	0
3	2	1	0

<u>50-60% A</u>

A Nominates	B Nominates 1	2	3
1	1	1	1
2	2	2	2
3	2	1	3

<u>25-40% A</u>

A Nominates	B Nominates 1	2	3
1	1	1	1
2	2	1	0
3	2	1	0

<u>60-75% A</u>

A Nominates	B Nominates 1	2	3
1	1	1	1
2	2	2	2
3	2	3	3

<u>40-50% A</u>

A Nominates	B Nominates 1	2	3
1	1	1	1
2	2	1	2
3	2	1	0

<u>75-100% A</u>

A Nominates	B Nominates 1	2	3
1	1	1	1
2	2	2	2
3	3	3	3

Take, for example, the matrix for the case in which A has 50-60% of the vote. If each party nominates only one candidate, both are elected, and the payoff to A is one. (This is an extremely rare event, occurring only when there is a strong third party—a case not covered by the present exposition. Technically, this feature makes the game non-zero-sum, in that the gains of A plus the gains of B do not sum to the constant three for this case. In a two party situation, however, this combination of strategies is never optimal, and, in practice, never occurs.) If A nominates one while B nominates two or three, A, being in the majority, will elect that one, while B will elect the other two. If A nominates two, he will elect both, regardless of how many B nominates. The third strategy of A presents an interesting inversion in outcomes: if B nominates one, he will elect that one and A will elect two; if B nominates two, he will elect both (despite his overall minority, each of his <u>two</u> will have <u>more</u> than 20% of the vote, while each of A's <u>three</u> candidates will have <u>less</u> than 20%); but if B nominates three, he will elect none and A will elect three. This latter case illustrates particularly how it is possible to nominate either too many <u>or</u> too few candidates.

Solution of Game

Taking the same payoff matrix (50-60%) as before, what is the optimal strategy for each party? Examining A's strategies, it is apparent that by nominating two, he can guarantee elcting that many, which is more than can be guaranteed by any other choice of strategy. B, on the other hand, who wishes to minimize the number elected by A, can assure by choosing to nominate two, that A will elect at most two. The same assurance is given B is he chooses to nominate only one, but this strategy is dominated, since the outcome resulting from its selection is in no case better, and sometimes worse than that from nominating two. Thus the minimax loss solution for this game matrix calls for each party to nominate two, in which case the outcome is that A selects two.

In the 60-75% game, B can, by nominating only one, guarantee himself electing that one, while for A, nominating either two or three guarantees his electing two. Nominating two, however, is consistent with his minimax strategy in the 50-60% case, and provides for the event that the vote should actually fall below 60% (which is much more likely, on the basis of the distribution of the vote, than that it will exceed 75%).

In the 75-100% game, A's best strategy clearly is to nominate three, which guarantees his electing all of them, while B can by no choice guarantee himself even one. It might seem that B's best choice were to nominate two or three, in the hope that A might grossly err by nominating only one. It seems more reasonable, to assume, however, that B in this case acts as if he knew the a priori distribution of A's choices to have zero probability for the strategy of nominating one. (In the nearly 1500 district elections since 1902, this has indeed been the case: with 75% of the vote, the majority party has never nominated less than two.) With this assumption, B's three strategies given identical outcomes and leave no choice. However, given even the slightest positive probability (which, in practice, there always is) of the percentage of the vote to party A being not 75-100 but between 60% and 75%, then B should nominate only one, as a hedge against this possibility. In addition, other considerations enter, such as the cost of candidacy, the effect of running superfluous candidates in the face of certain defeat, etc. It might be supposed in this situation that B should really choose to run no candidates. Again, however, there is always some positive probability that the 75-100% matrix is not the one which applies, but rather 60-75%, in which case a candidate should always be nominated. In addition, there may be a positive benefit to party morale to run one rather than none.

Solutions for the three remaining matrices are symmetrical to the ones already obtained, and, in summary, the hypothesized behavior is that the number of nominations by each party will conform to the table below:

% of the Vote to Party A	Nomination Pattern
0-25	1-3
25-40	1-2
40-50	2-2
50-60	2-2
60-75	2-1
75-100	3-1

Rationale for minimax solution. The conservative minimax loss criterion—providing a sure minimum rather than a chance of greater gain or loss—appears particularly compatible with a stable political system. Using the minimax loss solution assumes that parties do not try to completely demolish the opposition, even at considerable risk to themselves. One would not expect, on the other hand, such a consensus-promoting model to apply to revolutionary parties, who might be more likely to risk all for total victory.

Alternative Solutions. One might consider the minimax regret criterion, and ask, given a particular choice of strategy by the opponent, what the regret would be at one's own choice of strategy compared with one's best choice given that particular strategy of the opponent. While much post mortem speculation is carried on over election results, it is not clear that this neurotic criterion is invoked before the choice is made. One can compare, however, the solutions obtained by minimaxing loss with those obtained by minimaxing regret. In the latter case, of course, a non-zero-sum game results, since A's regret for a particular outcome depends upon the other entries in that row. In the case, however, of these small matrices and limited range of payoffs, it happens that precisely the same solutions result from minimaxing either regret or loss.

In general, solution of a game by the minimax principle assumes a rational opponent who will behave in the same conservative fashion, i.e., to assure himself a guaranteed himimum. Such behavior may not be optimal against an opponent who behaves in some other fashion, although it will still guarantee one the stated minimum. It may be more profitable yet, though, to utilize some more risky strategy. In particular, if some a priori distribution can be ascribed to the strategies of the opponent (as was done in a limited sense in the 0-25% matrix), then a Bayesian solution, given this particular distribution, can be made. A party nominating committee could act on such a basis, of course; the problem is in specifying the a priori distribution. The number of previous election upon which to base experience is small, and in most districts the distribution of the vote has varied considerably, affecting the choice of strategy. It seems unreasonable, therefore, to assume constant probabilities, unchanging over time. Yet if separate distributions among strategies are posited for different distributions of the vote, the number of previous elections upon which to base such a distribution becomes very small. In addition, committee membership, and perhaps party philosophy, is continually changing. Considering all these contingencies, minimax loss seems the most appropriate model for this situation.

Table 2

Distribution of Illinois General Assembly Elections 1902-1954, by Number Nominated and % Democratic

Number Nominated		% Democratic for the Same Election						
Dem	Rep	0-25	25-40	40-50	50-60	60-75	75-100	Total
1	3	27%	5%	1%				3%
1	2	69%	81%	26%	6%	2%		36%
2	2	4%	13%	68%	67%	10%		40½%
2	1		1%	5%	27%	87%	91%	20%
3	1					1%	9%	½%
Total		77	371	419	295	145	45	1353

Since the proportion of the vote received by party A is a variable, imperfectly estimated, it is not entirely certain, of course, that the vote will be within the range applicable to a particular matrix. Hence, as a first approximation to a stochastic model, the solutions just described incorporate the following feature: in the case of a matrix for which a party has two possible minimax strategies, that strategy is preferred which is consistent with the minimax strategy of one of the adjacent matrices (the one closer to the middle of the distribution). A further step might involve attaching a probability to the vote being in each of the six ranges, and solving the composite game, based upon the relations among the six probabilities. However, the present examination of empirical data, described in the next section, is based upon the simpler model.

ANALYSIS OF ELECTION DATA

To examine the fit of the model, data were obtained from official records (Illinois Secretary of State, 1902-1954) for the 1377 biennial elections (27 in each of Illinois' 51 districts) for the years 1902-1954. (Fortunately for research purposes —although in direct disregard of the constitution

Table 3

Partial Regression Coefficients for Number Nominated at Time t

Variable	Democrats	Republicans
Number nominated at time t-1	.462	.519
% Democratic at time t	.470	.301
% Democratic at time t-1	.012	.044
% Democratic at time t-2	-.085	.027
Minimax outcome at time t-1	.041	-.022
Multiple Correlation	.767	.800

Note: For the Republicans, the second, third, and fourth partial regression coefficients are for % Republican, at times t, t-1, and t-2.

—the Illinois Assembly had failed to redistrict during this entire period.)

Of these 1377 elections, 187 resulted in one party receiving less seats than was guaranteed it by following a minimax strategy. (Of the non-minimax outcomes 29 were, however, the result of uneven distribution of the party's vote among its candidates.) In 86 of the non-minimax outcome elections, the Democrats won less seats than guaranteed by minimax strategy; in 101 elections, the Republicans were the losers. The following sections further examine factors involved in rationality of the choice of the number nominated.

Number Nominated and Per Cent Democratic

Table 2 displays, for each of the six ranges of the vote which is Democratic (the same as for the six payoff matrices in Table 1), the distribution of the number nominated: 1-3 (one Democrat; three Republicans), 1-2, 2-2, 2-1, 3-1. Columns add to one hundred per cent. The less than two per cent of the elections (many of them involving strong third parties) which display a nomination pattern other than one of these five are omitted from the table.

From both sets of marginals, it is evident that Illinois has been more Republican than Democratic over this period. The table entries, themselves, however, show a remarkable symmetry between the two parties, for the more than ninety per cent of the elections in which the Democratic vote was between 25% and 75%. The second (25-40% Dem) column (5, 81, 13, 1, 0) corresponds closely to the reverse of the fifth (25-40% Rep) column (1, 87, 10, 2, 0); likewise the third column (1, 26, 68, 5, 0) to the reverse of the fourth (0, 27, 67, 6, 0). Given a vote between 25% and 75%, the parties behave rather similarly with respect to number nominated.

As an indication of the appropriateness of the particular game theory model in this situation, one may examine the distribution of number nominated for each of the six ranges of the vote. The predicted cells (as on the previous page) contain 27%, 81%, 68%, 67%, 87%, and 9% of the elections in their range of the vote, the overall proportion in those cells being 69%.

Table 4

Change in Number Nominated vs. Change in % Democratic

Change in Number Nominated	Change (±) in % Dem 0-10	10-50
Democrats		
Same direction as (Dem) vote	9%	26%
No change in number nominated	87%	72%
Opposite direction from vote	4%	2%
Republicans		
Same direction as (Rep) vote	7%	12%
No change in number nominated	91%	84%
Opposite direction from vote	2%	4%
Number of elections	1080	246

In over half of all elections, the vote is between 40% and 60% Democratic. In about two-thirds of these elections both parties employ the minimax strategy of nominating two candidates. In another quarter of these elections, only the majority party nominates two candidates, with the result that the election is uncontested, but its outcome is the same as that obtained by use of minimax strategy. For about five per cent of the elections, however, the majority party nominates only one candidate, thereby losing a seat it could have obtained.

The case in which one party has between 60% and 75% of the vote results in an uncontested election about five-sixths of the time. Only rarely (2% for Democrats; 1% for Republicans) does the party with this distinct majority fail to nominate the two candidates which it can surely elect.

With more than 75% of the vote, however, while both parties are considerably reluctant to nominate the three which that proportion enables them to elect, the Democrats have been much less likely to nominate three than the Republicans (9% compared with 27%).

There is, in the foregoing results, a strong implication of non-linear utility; parties never fail to run at least one candidate; 5% of the time when they could elect two, they fail to run the second candidate; but 80% of the time when they could elect three, they fail to nominate a third candidate. Such non-linear utility, of course, implies a non-zero-sum game.

Sensitivity to Change

In examining various aspects of the sensitivity of the behavior of the nominating committee to change, the most striking result is the basic conservatism of both parties. No matter what, by far the most likely number of nominations for any given year is the same as that for the previous election.

Table 5

Correlations over 51 Districts

	Variable	1	2	3	4	5	6	7	8	9	10	11
1.	Number of contested elections	—										
2.	Number of non-minimax losses: Dem	-.43	—									
3.	Number of non-minimax losses: Rep	-.40	-.17	—								
4.	Number of changes in No. Nom'd: Dem	.14	-.08	-.12	—							
5.	Number of changes in No. Nom'd: Rep	.01	.21	.05	-.03	—						
6.	Mean % Democratic	-.04	.67	-.41	-.23	.19	—					
7.	Absolute value of (Mean % Dem - 50%)	-.57	.24	.44	.11	-.07	-.28	—				
8.	Variance (over years) in % Dem	-.27	.43	.15	.04	.23	.10	.11	—			
9.	Change in % Democratic, 1902-1954	-.20	.62	-.06	-.11	.05	.42	.06	.70	—		
10.	Var of 2 yr change in % Democratic	-.42	.58	.20	.00	.19	.29	.17	.64	.59	—	
11.	Var of presid't'l yr change in % Dem	-.32	.41	-.02	.01	.17	.20	.03	.37	.34	.72	—
12.	Var of off-year change in % Dem	-.21	.31	.25	.18	.09	.09	.25	.59	.28	.54	.14

Changes in number nominated often represent changes in the vote of several years prior.

<u>Multiple regression on number nominated</u>. To investigate the association of variables with the choice of the number of candidates, multiple regressions were performed separately for Democrats and Republicans, using the 1275 elections of 1906-1954. Independent variables were number nominated at the previous election; Percentage of the vote at the present election, the previous election, and the next previous election; and the occurrence of a minimax outcome (rather than a non-minimax outcome) at the previous election.

The same general pattern of partial regression coefficients was obtained for both parties (Table 3); nearly all the weight is placed on the two variables, number nominated the previous election, and percentage of the vote for the present election. (The relevance of the latter variable may be taken as reflecting on the committee's use of estimates of the forthcoming vote—which correlates, however, .82 with the vote of the previous election.) Note, though, that while for Democrats, the two variables are weighted about equally, for the Republicans, the number nominated in the previous year has significantly ($p<.01$) more weight, an appropriately more conservative result. The correlations of the number nominated the present election with the number nominated the previous election, the percentage Democratic the present election, the previous election, and the next previous election, are all between .60 and .75, and in decreasing order as named, for both Republicans and Democrats.

<u>Number nominated vs. change in % Democratic</u>. Examination of the behavior of each party in the face of a changing vote gives additional insight into the differential role of conservatism as a correlate of choice. The following implications are drawn from Table 4: (1) change (in the number of nominations from that of the previous election) —for all values of change in the vote—comes slowly for both parties, but moreso for the Republicans, (2) Democrats show a larger reduction, between the small and large vote change situations, in the proportion of elections showing no change in number nominated, than do Republicans, (3) when the vote is changing over 10%, and a party <u>does</u> change its number of nominations, the Democrats are more likely to change in the proper direction, (4) for the Democrats, the change in the number nominated is much more likely to be in the appropriate direction if the change in the vote is large, (5) for the Republicans, however, the change in the number nominated is no more likely to be in the appropriate direction for a large change of the vote than for a small change. (The differences of statements (1) to (4) above are all significant at $p<.01$.)

Examination of particular years of great change in the vote further documents the conservatism. Given below are the total number of changes in number nominated, over all 51 districts for the years of 1920 and 1924 (when the Republicans should have been increasing number nominated) and 1932 (when the Democrats should have been doing the same).

	Democrats			Republicans		
	Up	Down	Same	Up	Down	Same
1920	1	8	42	5	1	45
1924	4	4	43	5	7	39
1932	8	1	42	0	7	44

Some anticipation is shown, but far from adequate: in 1920, the Republicans lost 21 seats they could otherwise have gained, through failing to run sufficient candidates; four years later they lost 14 seats in a similar fashion; in 1932, the Democrats lost 8 seats by failing to run enough candidates.

Table 6

Correlations over 25 years, 1906-1954

Variable	1	2	3	4	5	6	7	8	9	10	11
1. Year	—										
2. Number of contested elections	.42	—									
3. Number of non-minimax losses: Dem	.14	.38	—								
4. Number of non-minimax losses: Rep	-.25	-.07	-.26	—							
5. Number of changes in no. nom'd: Dem	-.36	-.58	-.36	.32	—						
6. Number of changes in no. nom'd: Rep	-.43	.24	.31	-.07	-.42	—					
7. Mean % Democratic	.45	.44	.71	-.69	-.54	.19	—				
8. Mean of 2 yr change in % Democratic	-.06	-.10	.27	-.67	-.34	.37	.44	—			
9. Mean of 4 yr change in % Democratic	-.06	.15	.35	-.40	-.44	.64	.44	.46	—		
10. Variance in % Democratic	.39	.20	-.03	.01	.10	-.17	-.08	.02	-.13	—	
11. Variance of 2 yr change in % Dem	-.74	-.05	-.12	.52	.38	.37	-.53	-.09	-.07	-.02	—
12. Variance of 4 yr change in % Dem	-.70	-.37	-.18	.04	.20	.30	-.31	.38	.00	-.18	.59

Number Nominated vs. Non-minimax outcome. A similar reluctance to change the number nominated can be observed even in the case in which the previous election resulted in a lesser number of seats than the minimax strategy would have guaranteed. For the Democrats, loss of a seat by occurrence of a non-minimax outcome at the previous election increases the probability of change in the number nominated from .15 to .26, while for the Republicans, the corresponding probabilities are .10 and .20. Following a non-minimax outcome, the Democrats and Republicans each have a probability of changing the number of nominations which is only about ten per cent greater than that obtaining if the number elected were at least equal to the number guaranteed by following minimax strategy. Of the changes in the latter situation, two-thirds are in the direction of nominating more candidates, which is appropriate, since practically all of the non-minimax losses result from nominating too few, rather than too many candidates.

Relations over Districts and Years

In the following three sections, relations are examined between a number of variables by (1) correlating them using the 51 districts as the units, (2) correlating them over the 25 years, 1906-1954, and (3) examining in further detail changes over time.

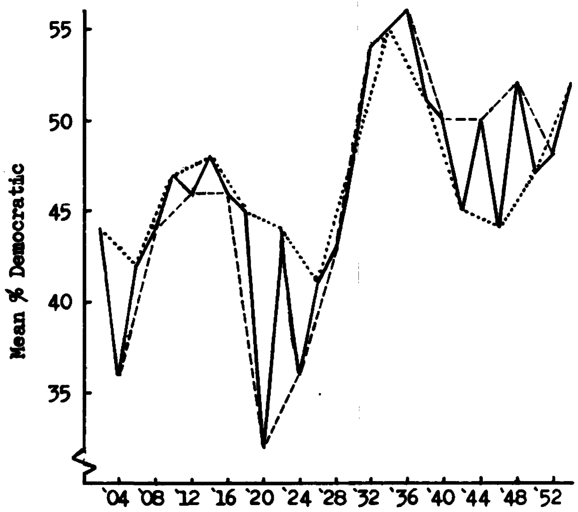
Correlations over districts. Table 5 presents correlations among 12 variables for the 51 districts. The variables represent the experience of each district over the period 1902-1954, and are as follows:

1. Number of elections in which there were a total of more than three candidates for the three seats.
2. Number of elections in which the Democrats elected fewer than guaranteed them by following the minimax strategy.
3. Number of elections in which the Republicans elected fewer than guaranteed them by following the minimax strategy.
4. Number of elections for which the number nominated by the Democrats represents a change from the previous election.
5. Number of elections for which the number nominated by the Republicans represents a change from the previous election.
6. Mean, over all years, of the variable, % Democratic.
7. Absolute value of the difference between 50% and variable (6) above.
8. Variance, over all years, of the variable, % Democratic.
9. Change in % Democratic from 1902 to 1954.
10. Variance of the change in % Democratic from the previous election.
11. Variance of the change in % Democratic from the second previous election, for the 12 presidential election years, 1908-1952.
12. Variance of the change in % Democratic from the second previous election, for the 13 non-presidential election years, 1906-1954.

Notice the relation of non-minimax losses to the other variables. Districts having more contested elections have fewer non-minimax outcomes, although even in these districts, far from all of the elections are contested. Perhaps the willingness to contest is the critical element. Each party has the most non-minimax losses in those districts in which its mean proportion of the vote is highest. This may again reflect the decreasing utility of the third seat. For the Democrats, non-minimax losses are also greater in those districts where the mean vote has been increasing over time. The number of contested elections is much greater

Figure 1

Mean % Democratic of all 51 Districts, 1902-1954



in those districts where the mean vote is close to 50%.

Correlation over years. Correlations were also computed (Table 6) among 12 variables for the 25 years, 1906-1954 (1902 and 1904 could not be used, since some of the variables involved changes from two or four years previous). For each year, the variables below represent all 51 districts. Variables 2-7, and 10 correspond to variables 1-6, and 8, respectively in the correlations over districts. The other variables are

1. Year of the election: 1906, 1908, ..., 1954;
8. Mean, over all districts, of the variable, change in % Democratic from the previous election.
9. Mean, over all districts, of the variable, change in % Democratic from the second previous election.
11. Variance, over all districts, of the variable, change in % Democratic from the previous election.
12. Variance, over all districts, of the variable, change in % Democratic from the second previous election.

The changes associated with time have been considerable in Illinois, and many significant relations emerge. Paralleling the previous finding concerning districts, non-minimax losses are highly related to the party strength that year, as well as to increase in the party's vote from two and four years previous. This is true for both parties. Both parties are also more likely to change the number of nomination from the previous year when their vote is dropping than when it is rising.

A number of variables change with time itself, there being an increase in the number of contested elections; decreases in the number of changes in number nominated from the previous election (for both parties); an increase in the Democratic vote; an increase in the variance among districts, and sharp decreases in the variance of the change from the previous election or the second previous election. Changes over time are further examined in the next section.

Changes in the vote, 1902-1954. Figure 1 gives the mean Democratic vote of the 51 counties for each of the 27 biennial elections, 1902-1954. (The solid line connects adjacent election years; the dashed line connects presidential years; the dotted line connects non-presidential years.)

It can be seen that the variance among the means of the districts for presidential years is much greater than that among off-years, in fact, less than half. However, the variance among districts within years is much less for presidential years than for off-years. Associated with presidential voting, then, is a unifying effect among districts. However, the unity among districts of the presidential years is less stable than the relative diversity among districts of the off-years. If the presidential election could be said to bring the districts together, then it tends to unite them at a different level from election to election, following which the districts return in the off-years to a more stable diversity.

In general, the variance among districts is greater than the variance among years, and the greater variance among districts is associated particularly with the division between Cook county (Chicago and suburbs) and the rest of the state. Moreover, this division has been increasing over time. In 1958, the average Democratic vote in the Cook county districts was 64%, compared to 47% for the remaining districts.

IMPLICATIONS

It is apparent from the preceeding sections that the particular game theory model employed provides a partial, though not a complete description of the behavior of the nominating committee in determining the number of candidates to run. This section examines some factors relevant to the agreement and disagreement of data and model.

Rationality

Game theory may be employed as a model of rational behavior, defined in terms of minimaxing a certain quantity. In this sense, a considerable amount of rationality was demonstrated by the nominating committees in their determination of the number of candidates: in 69% of all elections, both parties employed a minimax strategy.

It is, however, for both parties, a rationality associated with a basic conservatism. Some years often elapse from a change in the vote to the corresponding adjustment in the number nominated. There is some question whether the effective behavior which is shown results from a considered rationality, or simply from the conservatism of not changing the number nominated, which usually turns out to be the best thing to do. Or perhaps, more significantly, the general decision for conservatism has been made on a rational basis, seeing that this is generally effective.

Utility and Job Security

The principal departures from the model occur in the relatively rare cases in which one party has more than seventy-five per cent of the vote. In these cases, however, the departure is very considerable, with parties only nominating three candidates about 20% of the time. As has been previously suggested, there is a strong implication of non-linear utility particularly with respect to the election of third candidates. Further investigation has revealed some possible bases for this non-linear utility.

A relevant consideration seems to be the individual job security of the incumbent (or less often of another strong candidate). While parties may acknowledge that with 75% of the vote, they may elect three, and even if the vote falls somewhat below that, will surely elect two, the critical question to them is "Which two?" The party committee naturally seeks to control who is nominated, as well as the number nominated. Hence, coalitions may form between the two stronger forces within the party and mitigate against adding a third nomination which would reduce the certainty. (The possibility of coalitions of two against one would seem to make job security initiated pressure against an additional nomination more likely against a third candidate when the vote is 75%, than against a second candidate when the vote is 50%.)

In addition, there is evidence that in certain cases, again most likely occurring when one party has 75% of the vote, bi-partisan agreements are reached to allow the minority a seat, in exchange for an appropriate "side-payment." The bi-partisan agreement is also seen to alleviate the job security problem, and perhaps to satisfy both parties, inasmuch as, if the utilities are non-linear, the side payment can be larger than the utility of the majority's third member and less than the utility of the minority's first member.

Minority Representation

In terms of the intent of the law—to provide minority representation—the game theory solution is optimal. If both parties follow a minimax strategy, and divide their votes equally among all their candidates, the result will be to give the minority one seat whenever it has as much as 25% of the vote. This end has been generally achieved, as noted by Blair (1960). (Indeed, as we have seen nearly all of the mal-representation which has occurred has been in giving the minority over-representation.)

In the earlier days of cumulative voting in Illinois, some "good government" groups protested the lack of choice available to the voter in the general election—with many elections having only three candidates, no more than the number of seats —and hence proposed that parties be required to run full slates. This would, of course, negate the basic purpose of this scheme of minority representation. In addition, several writers have implied that a non-contested election represented collusion, but this is not necessarily so. Such a casy may indeed be the best strategy for each party, and may also represent the only way in which nearly proportional representation can be assured.

A modification might be proposed, however, to guarantee more proportional representation by removing the uncertainty which influences committees to unnecessarily (and unprofitably) limit the number of candidates. Allow the primary elections to specify the order of election of candidates, and the number of votes for the party (without regard to candidates) in the general election to determine the number elected from each party, in the already-specified order. Better yet, let one election suffice for both functions, by allowing the votes within a party to determine the ordering of candidates within that party, and the total number of votes cast for each party to determine the number of candidates it elects. Each voter would, as before, have a number of votes equal to the number of candidates to be elected, and would still be able to cross party lines if desired. Thus proportional representation could be achieved to a much greater extent, unhampered by possibly non-optimal committee decisions.

SUMMARY

A two-person zero-sum game theory model is devised for the behavior of committees which determine, for their own three-member districts, the number of candidates (one, two, or three) their party shall enter in the general election for representative to the Illinois General Assembly. Application of this model to the 1377 biennial elections from 1902 to 1954 finds that in 69% of all elections both parties employ minimax strategy, and that 86% of all elections result in a minimax outcome, regardless of strategies. Other results include (1) a drastically decreasing utility for the third seat, possibly related to individual job security, (2) a basic conservatism on the part of both parties, which are considerably reluctant to change the number nominated, and (3) changes in the distribution of the vote over the period 1902-1954.

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